ON ECONOMIC ANALYSIS OF LARGE SCALE COMPLEX SYSTEMS

Ramaz Khurodze
Member of Georgian National Academy of Science, Vice –President, Tbilisi, Georgia

Revaz Kakubava
Professor at Georgian Technical University, Tbilisi, Georgia

ABSTRACT

In the given paper large scale redundant complex system with renewable units is considered. Two maintenance operations – the renewal (repair) of any failed unit, and the replacement of failed main unit by redundant one) – are performed in the system. The numbers of main units as well as redundant ones, also the numbers of replacement servers as well as renewal ones are arbitrary. An economic criterion for dependability planning (structural control) of considered system is introduced; the optimization problem is stated and partially investigated.

1. INTRODUCTION

Economic analysis of large scale complex systems (computer and telecommunication networks, gas and oil pipelines, power systems, defense systems, transport systems, etc.) is one of the topical directions in the modern management science [1-6], [12-16]. This problem plays a key role in the dependability planning (structural control by economic criteria) of telecommunication and production systems [1], [4].

The object of our study is any large-scale territorially distributed redundant technical system and its structural control subsystem. The controlled system consists of unreliable, repairable units.

At the same time, telecommunication systems, and in particular, Mobile Communication Networks are the most relevant (the most suitable) field for applications of our research.

Let's consider the last statement in detail.

Since the beginning of the second half of the 80ies of the last century considerable changes have taken place in the field of telecommunication, affecting the interests of telecommunication service providers, as well as users and equipment manufacturers.

Here we mean de-monopolization in the field of telecommunication services in almost all developed countries of the world; creation of open market of traditional and new types of service and as a sequence, intensification of competition between telecommunication service providers; strengthening of requirements on the part of users; rapid development and inculcation of new technologies, architectures, etc.

The result of these changes is a considerable increase of interest to the problems of structural control and management of telecommunication networks and their components.

The essence of this is: low dependability threatens telecommunication service providers with not only possible loss of clients, but also directly affects the economical indices. The refuse of service to users, because of non-serviceability of telecommunication means, is expressed in lost income and frequently in direct losses caused by penal sanctions claimed by users.

Naturally, this problem was in the focus of ITU (International Telecommunications Union). Namely, ITU has issued recommendation E.862 "Dependability planning of telecommunication networks" (Geneva, 1992), which by ITU's assignment was prepared by Swedish experts.

Recommendation E.862 is concerned with models and methods for dependability planning, operation and maintenance of telecommunication networks and the application of these methods to various services in international network.

For all that, this Recommendation gives quite convincing preference to analytical methods for dependability planning, compared to other methods.

In particular, it is ascertained that the application of the analytical methods gives economically the best-balanced level of dependability, seen from the customer's point of view. This reduces the risk of customer's complaints and loss of business to competitors, as well as the risk of unnecessary investments. It is, therefore, considered to be the best general way of planning for administration, as well as for customers [1].

After this ITU has worked out a lot of documents on the problems of telecommunication service quality and dependability. According to ITU documents Quality of Service (QoS) is the degree of service a provider performs for a client, including the existing agreement relation between them (Service Level Agreement -SLA).

SLA between service providers and users regulates technical, organizational, legal and financial aspects and is becoming an important factor in competition in the developed countries.

A substantial part of SLA is the list of dependability indices and the values the guarantee of their provision being given by the provider. And this is not accidental, as dependability is one of the most important determining factor of quality.

Thus, service provider, i.e., company that leases telecommunication channels has to fundamentally study his possibilities, collect and process statistic data so that to have full idea about the quality of the provided service. Only in this way is it possible to rationally estimate the given guarantees, compensation forms and size he will have to pay in behalf of the client if SLA demands will be broken. In these conditions the optimization of dependability and structure of telecommunication networks and their components acquire particularly great importance.

2. SYSTEM DESCRIPTION

The technical system consists of ***m*** main and ***n*** redundant units. All units are identical. It is supposed that for the normal operation of the system, the serviceability of all ***m*** main units is desired. However, if their number is less than ***m***, then the system continues to function but with lower economic effectiveness. The main units fail with intensity α and the redundant ones – with intensity β. A failed main unit is replaced by a serviceable redundant one if there is such a possibility in the system. In the opposite case (all redundant units are non-serviceable, or the serviceable redundant units standby are already intended for the replacement of earlier failed main units) the replacement will be carried out as soon as it becomes possible. The failed units, both the main and the redundant ones, are repaired and become identical to the new ones. There are ***k*** replacement and ***l*** repair servers in the system. The time lengths of all operations in the system are random variables with various distributions. Thus, in a natural way we have a closed queuing systems (queuing systems with finite source) with two types of service – replacement and repair (renewal) (Fig. 1). On the other hand, when ***m*** is large number (in practice it might be tens, hundreds, thousands and more), we will suppose that we have infinite source of requests and will get open queuing systems.

3. ON ECONOMIC ANALYSIS

The system units as well as maintenance servers give different profits depending on their following states (positions).

For system units: 1. main; 2. serviceability redundant one being under replacement; 3. serviceability redundant not being under replacement; 4. failed redundant being under repair; 5. failed redundant not being under repair.

For maintenance servers: 6. working replacement; 7. non-working replacement; 8. working repair; 9. non-working repair.

The profit amount per some fixed time interval (time unit) from one system unit or one maintenance sever is c_i, $i=\overline{1,9}$ for above-mentioned states. Some of these c_i may be non-positive.

Denote by E_i, $i=\overline{1,9}$ the average number (mathematical expectation) of system units or maintenance servers at the arbitrary time moment in the steady state of system functioning.

The expressions for E_i, $i=\overline{1,9}$ are obtained in the works [4-21].

Namely,

$$E_1=\sum_{i=0}^{m}(m-i)\cdot\sum_{j=0}^{n+i}P(i,j);$$

$$E_2=\sum_{r=1}^{k-1}r\cdot\left(\sum_{j=0}^{n}P(r,j)+\sum_{i=r+1}^{m}P(i,\,n+i-r)\right)+k\cdot\sum_{i=k}^{m}\sum_{j=0}^{n+i-k}P(i,j);$$

$$E_3=\sum_{i=0}^{m}\sum_{j=0}^{n+1}(n+i-j)\cdot P(i,j)-E_2;$$

$$E_4=\sum_{r=0}^{l-1}r\cdot\sum_{i=0}^{m}P(i,\,r)+l\cdot\sum_{i=0}^{m}\sum_{i=l}^{n+i}P(i,j);$$

$$E_5=\sum_{i=0}^{m}\sum_{j=0}^{n+1}j\cdot P(i,j)-E_4; \qquad E_6=E_2;$$

$$E_7=k-E_6=k-E_2;$$

$$E_9=l-E_8=l-E_4;\; E_9=l-E_8=l-E_4;$$

We introduce the function for economic analysis of the system. This function expresses the profit of examined systems per time unit, taking into account the above-mentioned values.

Initial characteristics of considered systems – ***m***, ***n***, ***k***, ***l***, ***α***, ***β***, ***λ***, enter into the expression of profit function via $p(i,j)$ - probabilistic characteristics of considered systems. The expressions for them can be obtained in [4-21]. Profit function also depends on $\boldsymbol{c_1}$,… , $\boldsymbol{c_9}$ in the following way

$$F=F(n,k,l)=\sum_{i=1}^{9}c_iE_i$$
.

Eventually, the problem of system optimization is stated as problem of mathematical programming (integer programming).

Namely, with those fixed ***m***, ***α***, ***β***, ***λ***, ***μ***, $\boldsymbol{c_1}$,…, $\boldsymbol{c_9}$ for the considered system to select such values of the parameters ***n***, ***k***, ***l*** (optimal numbers of redundant units, replacement servers and renewal servers) so that the profit function **F** would accept maximum value and to determine this value.

That means the solving of problem of analytical synthesis of multi-unit recoverable redundant system by economical criterion.

We believe, this result will be very useful for experts working in the field of design, control and management of complex systems.

2.3. CONCLUSIONS

Main goal of mathematical model, constructed by us, is structural optimization (structural management) of considered system by economic criteria. Since connections between system's units is fixed, its structural optimization means selection of optimal quantity of redundant units maintenance servers.

From this point of view, intermediate characteristic values p (i, j), introduced by us, are very fruitful: 1) using them it is not difficult to construct mathematical model; 2) system's effectiveness criterion F (n, k, l) is easily expressed by their means.

Finally, structural control and management problem is brought to linear integer programming problem, which's solution is only computational difficulty.

REFERENCES

1. ITU-T Recommendation E.862 (Rev. 1). 1992. Dependability Planning of Telecommunication Networks, Geneva.

2. Korolyuk V.S., Limnios N. 2005. Stochastic Systems in Merging Phase Space. World Scientific, Imperial College Press.

3. Limnios N., Oprisan G. 2001. Semi-Markov Processes and Reliability. Birkhauser, Boston.

4. Ushakov I. 2006. Reliability: past, present, future. Reliability: Theory & Applications, vol. 1, no. 1, pp. 10-16 (electronic).

5. Khurodze R, Kakubava R, Saghinadze T, Svanidze N. (2022) An Alternative Transient Solution for Gaver's Parallel System with Repair, Bull. Georg. Natl. Acad. Sci. Vol. 16, N 4.

6. Khurodze R (2021) Markov Renewal Type Stochastic Model for Structural Control of Complex Systems, Bull. Georg. Natl. Acad. Sci., Vo. 15, N2

7. Khurodze R., Pipia G., Svanidze N. (2020) New Transient Solutions for some Semi-Markov Reliability Models. Bull. Georg. Natl. Acad. Sci. Vol. 14, N 4

8. Khurodze R. A. (2000) Reliability of Analog Devices for Control Systems(in Russian). Moscow, Nauka

9. Khurodze Ramaz, Svanidze Nino MATHEMATICAL MODELS TO CONTROL RELIABILITY FOR REDUNDANT TECHNICAL SYSTEMS, Georgian National Academy of Sciences, 2019

10. Kakubava R. (2020-2021) An alternative transient solution for semi-Markov queuing systems. Georgian Math.J.2021; 28(1), https://doi.org/10.1515/gmj-2020-2050, March 19, 2020

11. Kakubava, R.V., Sztrik, J., Svanidze N. (2019) Probabilistic analysis of a redundant repairable system with two service operations. J. Math Sci. Vol. 237, No. 8

12. Kakubava R.V. (2013) Reliability Model for Redundant System with Replacement Delays of Failed Elements. Automatic Control and Computer Sciences, 2013, Vol. 47, No. 2

13. Kakubava R. (2010) Multi Line Closed Queuing System for Two Maintenance Operations. RT&A. Vol.1 №1.http://www.gnedenko-forum.org/Journal/index.html

14. Kakubava R., Khurodze R.(2004) Technical Systems with Structural and Time Redundancy: A Probabilistic Analysis of their Perfomance. Automation and Remote Control Vol. 65, No 5, Plenum Publishing corporation, NY

15. Kakubava R. (2001) Analysis of queues under batch service in an M/G/1 system in a random environment. Automation and Remote Control. Vol.62, No.5, Plenum Publishing corporation, NY

16. Kakubava R., Khurodze R. (2000) Probabilistic analysis of the downtime of a duplicated system with recovery and switching. Automation and Remote Control. Vo. 61, No.9, Plenum Publishing corporation

17. Kakubava R., Khurodze R. 2004. Technical Systems with Structural and Time Redundancy: A Probabilistic Analysis of Their Performance. Automation and Remote Control, vol. 65, no. 5, pp. 825-833 (translated from Avtomatika I Telemekhanika, 2004, no. 5, pp.154-165)