ANALYSIS OF SOME ASPECTS OF CONSUMER BEHAVIOR IN THE SERVICE SPACE

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ABSTRACT

This study examines consumer behavior within service spaces, focusing on how individuals allocate limited resources to optimize utility in the consumption of various services. The research employs utility theory to construct mathematical models that elucidate decision-making processes, including the selection of an optimal service set based on prices, income levels, and individual preferences. The theoretical framework incorporates key principles of utility functions, marginal utility, and substitution rates, allowing for a robust analysis of consumer preferences and trade-offs. A case study using an additive utility function is presented to demonstrate practical applications of the model, revealing the alignment of theoretical constructs with real-world consumer choices. The findings contribute to the broader discourse on consumer behavior and provide actionable insights for designing service systems that maximize consumer satisfaction and operational efficiency.

Keywords: Consumer behavior, service spaces, utility theory, resource allocation, decision-making models, utility functions, marginal utility, substitution rates, consumer preferences, trade-offs, service optimization, mathematical modeling, consumer satisfaction, operational efficiency, additive utility function.

INTRODUCTION

One of the important concepts in economic theory is the consumer of goods and services. Specifically, the main problem in studying consumer behavior is to determine the quantities of services they will purchase at given prices and incomes, and how they will behave after the introduction of new services or technologies.

Moreover, when studying the problems of building and operating service systems, one must always consider the goals, desires, and needs of both the consumers and those who manage or are affected by these systems. Therefore, the study of utility represents an integral part of applied systems research at all stages of the life cycle of service systems (information, communication, etc.).

In this paper, we use the methods of utility theory [3] to construct and study models of consumer behavior to determine the optimal set of services when the consumer's capital over the considered period is limited to a given amount.

UTILITY FUNCTION AND CONSUMER BEHAVIOR MODEL FOR SOME SERVICES

The specific decision of a consumer to purchase a certain set of services can mathematically be represented as the selection of a specific point in the service space. Let n be the finite number of services considered, and $x=(x1...xn)$ the vector of services purchased by the consumer over a certain period (e.g., a year) at given prices and a certain amount of capital spent on these services over the same period. The service space is the set of all possible sets of services x with non-negative coordinates

$$C=\{x: x_i \geq 0\}.$$

In utility theory, it is assumed that each consumer initially has their own preferences on a certain subset of the service space $X = \{x: x_i \geq 0\}$. This means that for each pair $x, y \in X$ one of the three relations holds:

$x \succ y$ - set x is preferable to y;

$x \prec y$ - set x is less preferable than y;

$x \sim y$ - for the consumer, both sets have the same degree of preference.

Preference relations possess at least the following properties:

1) If $x \succ y$ and $y \succ z$, then $x \succ z$ (transitivity);

2) If $x_i > y_i$ for each i, then $x \succ y$ (non-satiation: a larger set is always preferable to a smaller one).

Under certain weak assumptions (the Debreu theorem: if the set X is connected and the preference relation is continuous, then a utility function exists [6]) concerning preferences, preference relations can be represented in the form of a preference indicator, i.e., such a utility function $u(x)$ that from $x \succ y$ follows $u(x) > u(y)$ and from $x \sim y$ follows $u(x) = u(y)$.

The introduction of the utility function allows replacing preference relations with the usual relations between numbers: greater, less, equal [3].

In utility theory, it is often assumed that the utility function is twice differentiable, strictly concave, and has the following properties:

1) With the growth of consumption of the good, utility increases;

2) A small increase in the good when it is initially absent sharply increases utility;

3) With the growth of consumption of the good, the rate of utility growth slows down;

4) With a very large volume of the good, its further increase does not lead to an increase in utility.

Recall that the function $u(x)$ is called strictly concave if $u(\alpha x + \beta y) > \alpha u(x) + \beta u(y)$ for any $x, y \in X$, $x \neq y$, $\alpha > 0$, $\beta > 0$, $\alpha + \beta = 1$.

Let $i, j = 1, 2, \ldots , n$. The matrix $(u_{ij}(x))$ is called the Hessian matrix.

We state without proof [3] the following assertion, which allows verifying the strict concavity of the function.

Assertion of strict concavity.

The function $u(x)$ is strictly concave if and only if

$$\sum_{i, j=1}^{n} \xi_i \xi_j u_{ij}(x) < 0$$

for all $\xi_i \neq 0$.

If the last inequality is satisfied, the matrix $(u_{ij}(x))$ is said to be negatively definite.

The marginal utility of a service is defined as the limit of the ratio of the utility increment to the increment of the service that caused this increase:

$$MU_i = \lim_{\Delta x_i \to 0} \frac{\Delta u(x)}{\Delta x_i}$$

Thus, marginal utility shows how much utility will increase if the volume of the service increases by a small unit.

The indifference surface is a hypersurface of dimension $(n-1)$ on which utility is constant:

$$u(x) = C = const$$

or in differential form

$$\sum_{i=1}^{n} \frac{\partial u(x)}{\partial x_i} dx_i = 0$$

(1)

Condition (1) means that the tangent to the indifference surface is perpendicular to the utility gradient [3].

From the consumer's point of view, the presence of a set of service sets with the same utility (i.e., the same degree of preference) means the possibility of replacing one set with another equivalent set, including the possibility of replacing one service with another.

Let in the ratio $dx_i = 0$ for $i = 3, \dots , n$, then this ratio takes the form

$$\frac{dx_1}{dx_2} = -\frac{\frac{\partial u(x)}{\partial x_2}}{\frac{\partial u(x)}{\partial x_1}} \tag{2}$$

That is, the marginal rate of substitution of the first service for the second is equal to the ratio of the marginal utilities of the first and second services. The substitution rate shows how many units of the second service are needed to replace the small unit of the first service.

The budget set is the set of those service sets that a consumer can purchase with capital k:

$$B = \{x: p \cdot x \leq k\}$$

where $p = (p_1, \dots , p_n)$ is the price vector, and $p \cdot x$ denotes the scalar product of vectors p and x.

In utility theory, it is assumed that the consumer always strives to maximize their utility, and the only thing that constrains them is the limited capital:

$$\max u(x) \text{ subject to } p \cdot x \leq k \tag{3}$$

In problem (3), it is assumed that the point of maximum $x^* \in X$. This constrained extremum problem is reduced to finding the unconditional extremum of the Lagrangian function - the Lagrangian [6].

We state without proof the general assertion for finding the conditional extremum of the function $u(x)$ with unknowns $x_1, \dots , x_n$ when the conditions include only the equations $v_i(x) = 0$; [6].

Assertion on the conditional extremum. Compose the Lagrangian

$$L(x, \lambda) = \lambda_0 u(x) + \lambda_1 v_1(x) + \lambda_2 v_2(x) + \dots + \lambda_m v_m(x)$$

where $\lambda_0, \dots , \lambda_m$ are some scalar values. For the maximum of the function $u(x)$ in the region $\{x_i: v_i(x) = 0\}$ to be achieved at point $x^* \in B \cap X$, it is necessary that there exist $\lambda_0, \dots , \lambda_m$, not all equal to zero simultaneously, such that they together with x^* satisfy the system of equations

$$\frac{\partial L(x, \lambda)}{\partial x_i} = 0, \quad i = 1, \dots , n; \quad v_i(x) = 0$$

At the same time, it is assumed that the functions $u(x)$ and $v_i(x)$ are differentiable at point x^*. If, in addition, the regularity condition is satisfied

$$\text{rank} \left(\frac{\partial v_i(x)}{\partial x_j} \right) = m \text{ at } x = x^*$$

then there certainly exist $\lambda_0, \dots , \lambda_m$, with $\lambda_0 \neq 0$.

We note that the given necessary conditions are homogeneous with respect to λ_i and actually define only the ratio of the components of the vector $(\lambda_0, \dots , \lambda_m)$ to any one that is not zero.

In our case, the regularity condition is satisfied, so we can set $\lambda_0 = 1$ and $\lambda_1 = \lambda$.

Then

$$L(x, \lambda) = u(x) - \lambda(p \cdot x - k).$$

The necessary conditions for a local extremum are:

$$\frac{\partial u(x)}{\partial x_i} - \lambda p_i = 0, \quad i = 1, \dots , n; \quad p \cdot x = k$$

(4)

(5)

If we require that $p > 0$, it can be proved that these conditions indeed determine the unique maximum point of the function $L(x, \lambda)$, which we denote as $(x^*, \lambda^*) = (x_1^*, x_2^*, \dots , x_n^*, \lambda^*)$. In particular, from (5) it follows that $x^* > 0$.

From (5) it can be seen that the consumer, with fixed income, chooses the set x^* such that at this point the ratios of the marginal utilities are equal to the price ratios:

$$\frac{\partial u(x)}{\partial x_i} / \frac{\partial u(x)}{\partial x_j} = p_i / p_j$$

Solving (4) and (5) with respect to x, we obtain the unique demand function of the consumer

$$x^* = x^*(p, k).$$

(6)

Namely, $x^* = (x_1^*, x_2^*, ... , x_n^*)$ is the set of services purchased by the consumer with capital K over the considered period, which maximizes their utility function.

To illustrate the theoretical conclusions obtained, we consider a particular type of additive utility function presented in the introduction for $n = 2$.

$$u(x) = a_1 \ln x_1 + a_2 \ln x_2, \quad a_1 > 0, a_2 > 0$$

It is easy to verify that this function satisfies all the previously stated requirements for utility functions.

Equations (4) and (5) for this function have the form

$$\frac{\partial u(x)}{\partial x_1} = \frac{a_1}{x_1} - \lambda p_1 = 0; \quad \frac{\partial u(x)}{\partial x_2} = \frac{a_2}{x_2} - \lambda p_2 = 0; \quad p \cdot x = k$$

(7)

Solving system (7) is straightforward. In particular, from the second and third equations, x_2 is easily expressed through x_1 (or vice versa), and then, after substituting it into the first equation, we obtain

$$x_1 = \frac{a_1 k}{a_1 p_1 + a_2 p_2}$$

$$x_2 = \frac{a_2 k}{a_1 p_1 + a_2 p_2}$$

(8)

(9)

The quantities (8) and (9) represent the components of the vector demand function $x^*(p, k)$.

Let $p_1 = 3$ currency units per unit of service, $p_2 = 4$ currency units per unit of service, $k = 60$ currency units. Substituting these numerical values of the parameters into (8) and (9), we get

$$x_1^* = 12$$

$$x_2^* = 10$$

CONCLUSION

This paper provides a rigorous exploration of consumer behavior in service spaces through the lens of utility theory. By deriving and analyzing mathematical models, the study highlights the interplay between consumer preferences, budget constraints, and service attributes. The theoretical insights and illustrative examples presented herein offer valuable implications for service system design and optimization, paving the way for future research in consumer-oriented economic modeling

ACKNOWLEDGMENT

"This project was carried out with the financial support of the National Science Foundation of Georgia (Grant ##GNSF/STO07/3-173). Any opinion expressed in the publication belongs to the author and may not reflect the views of the National Science Foundation of Georgia."

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